

0.1. Details on the proof of Theorem 7.

We explicate the details of one step in the argument for Theorem 7 in response to receiving several questions about it. Lines 2 through 4 of page 428, in the chain of inequalities in the proof of Theorem 7, involve the inequality,

$$\sum_{\ell=0}^{L-1} \mu(T^{-\ell-v_n}(B) \cap B) \leq (1 - \delta\delta_1)\mu(B).$$

Now, $T^{-\ell-v_n}(B) = B_{-\ell-v_n \pmod L}$ ranges over B_0, \dots, B_{L-1} . Let ℓ' denote the value in \mathbb{Z}_L such that $-\ell' - v_n \pmod L = 0$.

Recall from the proof that we have $\rho_n(\ell) \leq (1 - \delta_1)$ for all $\ell \in \mathbb{Z}_L$ and $\mu(B_\ell \cap B) \leq (1 - \delta)\mu(B)$ for all $\ell \neq 0$. The construction of ρ_n also gives that $\sum_{\ell=0}^{L-1} \rho_n(\ell) = 1$. So,

$$\begin{aligned} & \sum_{\ell=0}^{L-1} \rho_n(\ell) \mu(T^{-\ell-v_n}(B) \cap B) \\ &= \sum_{\ell=0}^{L-1} \rho_n(\ell) \mu(B_{-\ell-v_n \pmod L} \cap B) \\ &= \rho_n(\ell') \mu(B_0 \cap B) + \sum_{\ell=0, \ell \neq \ell'}^{L-1} \rho_n(\ell) \mu(B_{-\ell-v_n \pmod L} \cap B) \\ &\leq \rho_n(\ell') \mu(B) + (1 - \rho_n(\ell'))(1 - \delta)\mu(B) \\ &\leq \max \{ (1 - \delta_1)\mu(B) + (1 - (1 - \delta_1))(1 - \delta)\mu(B), (1 - \delta)\mu(B) \} \\ &= \max \{ (1 - \delta\delta_1)\mu(B), (1 - \delta)\mu(B) \} \\ &= (1 - \delta\delta_1)\mu(B) \end{aligned}$$

and the proof then continues as written.