

0.1. Restricted Growth on Polynomial Staircases.

Here we present the restricted growth condition specifically for polynomial staircase transformations. Let $\{s_{n,j}\}_{\{r_n\}}$ be the spacer sequence for a polynomial staircase transformation with defining polynomials p_n of degree less than or equal to a fixed positive integer D and with coefficients uniformly bounded by $Q < \infty$. What we have stated is that

$$s_{n,j} = p_n(j) = \sum_{a=0}^D c_{n,a} j^a$$

where $c_{n,a} \leq Q$ are the coefficients.

Consider the partial sums of the spacer sequence, letting k be any appropriate value:

$$s_{n,j}^{(k)} = \sum_{z=0}^{k-1} s_{n,j+z} = \sum_{z=0}^{k-1} p_n(j+z) = \sum_{z=0}^{k-1} \sum_{a=0}^D c_{n,a} (j+z)^a.$$

Now, $c_{n,a} \leq Q$ and $a \leq D$, so

$$s_{n,j}^{(k)} \leq DQ \sum_{z=0}^{k-1} (j+z)^D.$$

Since $j+k < r_n$,

$$s_{n,j}^{(k)} \leq DQ \sum_{z=0}^{r_n-1} z^D \leq DQ r_n^{D+1}.$$

Thus, if $\frac{r_n^{D+1}}{h_n} \rightarrow 0$ as $n \rightarrow \infty$ then $\frac{1}{h_n} s_{n,j}^{(k)} \rightarrow 0$ as $n \rightarrow \infty$ uniformly over j and k . Hence we conclude that the property

$$\frac{r_n^{D+1}}{h_n} \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

implies restricted growth for polynomial staircase transformations.

The staircase transformations correspond to the case when $D = 1$ which gives us precisely Adams' condition that $\frac{r_n^2}{h_n} \rightarrow 0$ as $n \rightarrow \infty$ for staircases to be mixing.