0.1. Partially Mixing and Partially Rigid Sequences. We present a class of examples of transformations defined by treating part of the stack as an odometer and part of the stack as a staircase in terms of the spacers. Let α be a real number between 0 and 1. Define a transformation T by cutting and stacking using the spacer sequence $\{s_{n,j}\}_{\{r_n\}}$ given by $s_{n,j} = j$ for $0 \leq j < \lfloor \alpha r_n \rfloor$ and $s_{n,j} = 0$ for $j \geq \lfloor \alpha r_n \rfloor$. T is then a staircase on the left α of the stack and an odometer on the right $1 - \alpha$ of it.

Now, consider T^{h_n} applied to a level $I_{n,i}$. On the left αr_n sublevels, it behaves as the staircase does but on the right $(1-\alpha) r_n$ sublevels it behaves as an odometer. Specifically, outside the rightmost subcolumn,

$$\mu(T^{h_n}(I_{n,i}) \cap I_{n,\ell}) = \frac{1}{r_n} \sum_{j=0}^{\lfloor \alpha r_n \rfloor - 1} \mu(T^{-j}(I_{n,i}) \cap I_{n,\ell}) + \frac{r_n - \lfloor \alpha r_n \rfloor}{r_n} \mu(I_{n,i} \cap I_{n,\ell}).$$

Let A and B be unions of levels. The above then yields

$$\mu(T^{h_n}(A) \cap B) = \frac{\lfloor \alpha r_n \rfloor}{r_n} \frac{1}{\lfloor \alpha r_n \rfloor} \sum_{j=0}^{\lfloor \alpha r_n \rfloor - 1} \mu(T^{-j}(A) \cap B) + (1 - \frac{\lfloor \alpha r_n \rfloor}{r_n}) \mu(A \cap B).$$

Hence, using the ergodicity of T and that $\mu(A \cap B) \geq 0$,

$$\liminf_{n \to \infty} \mu(T^{h_n}(A) \cap B) \geq \alpha \mu(A) \mu(B).$$

Similarly,

$$\lim_{n \to \infty} \mu(T^{h_n}(A) \cap A) = \alpha \mu(A) \mu(A) + (1 - \alpha) \mu(A) \geq (1 - \alpha) \mu(A).$$

Thus, we have shown that $\{h_n\}$ is α-partially mixing with respect to T and $(1-\alpha)$-partially rigid with respect to T. And so, T is at least $(1-\alpha)$-partially rigid.

Investigating the behavior or $T^{k_n h_n}$ for arbitrary $\{k_n\}$ is left to the reader.