Rules for Computing Limits

Math 31B — Darren Creutz

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To evaluate $\lim_{x \to a} F(x)$ for $a \in [-\infty, \infty]$ (possibly one-sided)

$\mathbf{F(a)}$	**Method**	$\lim_{\mathbf{x \to a}} \mathbf{F(x)}$
#	Direct	#
$\frac{0}{0}$	L'Hopital	$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$
$\frac{\pm\infty}{\pm\infty}$	L'Hopital	$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$
$\frac{\pm\infty}{\#}$	Direct	$\pm\infty$
$\frac{\#}{\pm\infty}$	Direct	0
$0(\infty)$	$f(x)g(x) = \frac{f(x)}{\frac{1}{g(x)}}$	$\lim_{x \to a} f(x)g(x) = \lim_{x \to a} \frac{f'(x)}{\left(g(x)^{-1}\right)'}$
0^0	$\ln((f(x))^{g(x)}) = g(x)\ln(f(x))$	$\lim_{x \to a} (f(x))^{g(x)} = e^{\lim_{x \to a} g(x)\ln(f(x))}$
0^∞	$\ln((f(x))^{g(x)}) = g(x)\ln(f(x))$	$\lim_{x \to a} (f(x))^{g(x)} = e^{\lim_{x \to a} g(x)\ln(f(x))}$
∞^0	$\ln((f(x))^{g(x)}) = g(x)\ln(f(x))$	$\lim_{x \to a} (f(x))^{g(x)} = e^{\lim_{x \to a} g(x)\ln(f(x))}$
∞^∞	Direct	∞

F(a)	**Example**
#	$\lim_{x\to 1} \frac{x^2+1}{2x+1} = \frac{1^1+1}{2(1)+1} = \frac{2}{3}$
$\frac{0}{0}$	$\lim_{x\to 0} \frac{\sin(x)}{x} = \lim_{x\to 0} \frac{(\sin(x))'}{x'} = \lim_{x\to 0} \frac{\cos(x)}{1} = \cos(0) = 1$
$\frac{\pm\infty}{\pm\infty}$	$\lim_{x\to\infty} \frac{3x+1}{x-1} = \lim_{x\to\infty} \frac{(3x+1)'}{(x-1)'} = \lim_{x\to\infty} \frac{3}{1} = 3$
$\frac{\pm\infty}{\#}$	$\lim_{x\to\infty} \frac{x+3}{6} = \frac{\infty}{6} = \infty$
$\frac{\#}{\pm\infty}$	$\lim_{x\to 3^+} \frac{x+4}{\ln(x-3)} = \frac{3+4}{\ln(0)} = \frac{7}{-\infty} = 0$
$0(\infty)$	$\lim_{x\to 0^+} x\ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{x^{-1}}$ $= \lim_{x\to 0^+} \frac{(\ln(x))'}{(x^{-1})'} = \lim_{x\to 0^+} \frac{x^{-1}}{-x^{-2}} = \lim_{x\to 0^+} -x = 0$
0^0	$\lim_{x\to 0^+} x^x = \lim_{x\to 0^+} e^{\ln(x^x)} = e^{\lim_{x\to 0^+} x\ln(x)} = e^0 = 1$
0^∞	$\lim_{x\to 0^+} x^{\ln(x)} = e^{\lim_{x\to 0^+} \ln(x)\ln(x)} = e^{\lim_{x\to 0^+} (\ln(x))^2} = e^{(-\infty)^2} = e^\infty = \infty$
∞^0	$\lim_{x\to 0} \left(\frac{1}{x}\right)^x = e^{\lim_{x\to 0} x\ln(\frac{1}{x})}$ and $\lim_{x\to 0} \frac{\ln(x^{-1})}{x^{-1}} = \lim_{x\to 0} \frac{(\ln(x^{-1}))'}{(x^{-1})'} = \lim_{x\to 0} \frac{\frac{1}{x^{-1}}(-x^{-2})}{-x^{-2}} = \lim_{x\to 0} x = 0$ and therefore $\lim_{x\to 0} \left(\frac{1}{x}\right)^x = e^0 = 1$
∞^∞	$\lim_{x\to\infty} x^x = e^{\lim_{x\to\infty} x\ln(x)} = e^{\infty(\infty)} = e^\infty = \infty$